Mathematical Foundations for Financial Engineering
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Objectives:
The main objective of this class is to prepare students for more quantitative classes in the MFE program. This course is a review of basic mathematical concepts and techniques, carefully selected for their relevance to financial applications. The material is preseted with a strong finance focus, emphasizing the subtleties and nuances specific to practical mathematical finance. Comprehensive notes will be handed out. There is intensive and finance-oriented homework.

Reference textbooks (recommended):
1. Numerical recipes in C (or C++) by Press et al. (Cambridge University Press.)
2. Introduction to calculus and analysis, Vol. I and II by Courant and John (John Wiley and Sons.)

Fundamentals
1. Sets and functions
2. Sequences and series
3. Dimensional analysis

Applications:
Discounting modeling
Mortgage pricing
Insuring consitency in financial mathematics

One-dimensional calculus
1. The role of continuity in finance
2. Mathematical derivatives and the chain rule
3. Order of magnitude and the Taylor expansion
4. The Rieman integral and the Leibnitz rule

Applications:
Hedging analysis and replication error tracking
Price sensitivity analysis of financial contracts

Multidimensional calculus
1. Description of multidimensional functions
2. Partial derivatives
3. Multidimensional integrals
4. Multidimensional series expansions

Applications:
Equity options sensitivity analysis
Connecting exotics and vanilla options

Differential equations
1. Ordinary differential equations: Classification and elementary solution techniques.
2. Partial differential equations: Origin and classification
3. The convection-diffussion partial differential equation and its relevance to finance

Applications:
The PDEs of financial pricing, their origin and properties
Setting up the pricing framework for complex financial contracts

Linear algebra
1. Algebraic linea equations: Independence and colinearity and their relevance to finance.
2. Matrices: Eigenvalues and eigenvectors and their importance in finance

3. Main numerical solution techniques for linear systems.
4. Tridiagonal linear systems

Applications:

Cash flow modeling problems

Stochastics for financial applications

1. Random variables and probability measures in finance
2. Covariance matrices and their properties
3. Asset pricing fundamentals
4. Stochastic calculus for finance in a nutshell

Applications:

The fundamental equity price process, its properties and transformations
Characterizing portfolio returns

Introduction to simulation in Finance

1. Point estimators and their simulation
2. Confidence interval estimations

Applications:

Monte Carlo pricing of European options
Value at risk analysis of portfolios

Introduction to numerical methods for non-linear systems and optimization

1. One-dimensinonal and multi-dimensional root finding techniques
2. Numerical integration methods
3. Interpolation techniques
4. Lagrange multipliers

Applications:

Implied volatility calculations
Yield curve modeling
Numerical integration in option pricing
Portfolio optimization